Homeworks for $8^{th} - 9^{th}$ weeks

Find a parameterization of a curve given by equation

1. (a) $x^2 + 10x + y^2 - 8y = -16$
   (b) $x^2 + 2y^2 - 2x + 12y + 18 = 0$
   (c) $x \ln y = 1,\ y > 0$
2. (a) $x^2 + 4x + y^2 - 4y = -4$
   (b) $4x^2 + 9y^2 - 8x - 36y + 4 = 0$
   (c) $y\ e^x = 2$
3. (a) $x^2 + 4x + y^2 - 2y = -4$
   (b) $4x^2 + y^2 - 4x = 0$
   (c) $\sqrt[5]{y}\ x = -1$
4. (a) $x^2 + 2x + y^2 - 6y = -2$
   (b) $4x^2 + 9y^2 - 16x - 18y + 24 = 0$
   (c) $\frac{y}{\cos x} = -0{,}5$

Sketch a graph of a curve $\mathcal{K}$ given by parameterization φ. Find a tangent vector at point T and draw it to the same picture as $\mathcal{K}$.

1. $\varphi(t) = (\ln t, \frac{1}{t}),\ t \in (0, \infty),\ T = (1, ?)$
2. $\varphi(t) = (e^{2t}, 2t - 1),\ t \in \mathbb{R},\ T = (?, -1)$
3. $\varphi(t) = (\cos 3t, 3t + \frac{\pi}{2}),\ t \in \mathbb{R},\ T = (?, -\frac{\pi}{2})$

Prove that function F is an antiderivative of function f on interval I

$$F(x) = \ln^2 x,\ \ f(x) = \frac{2\ln x}{x},\ \ I = (0, \infty).$$

Find antiderivatives of the following functions

(a) $6x^3 + \sqrt{2x^3} + \frac{1}{x})$
(b) e^{2x+5}
(c) $\cos(-x + \frac{\pi}{3})$
(d) $\sqrt[3]{3x - 1}$

Compute

(a) $\int \frac{x^2}{x^2+1}\,\mathrm{d}x$
(b) $\int \cos^2 x\,\mathrm{d}x$
(c) $\int \frac{(1+x)^2}{\sqrt{x^3}}\,\mathrm{d}x$
(d) $\int \frac{3+xe^x}{x}\,\mathrm{d}x$
(e) $\int \sqrt{\frac{9}{1-x^2}}\,\mathrm{d}x$

Compute following definite integrals

(a) $\int_{\frac{\pi}{2}}^{\pi} (\sin 2x + x)\,\mathrm{d}x$
(b) $\int_{-1}^{1} (x^2 - x + 2)\,\mathrm{d}x$

Compute

(a) $\int \sqrt[5]{1-3x}\,\mathrm{d}x$
(b) $\int \frac{x}{2-x^2}\,\mathrm{d}x$
(c) $\int \frac{x^3}{\sqrt[4]{(1-3x^4)^3}}\,\mathrm{d}x$
(d) $\int \sqrt{\frac{1-\sqrt{z}}{z}}\,\mathrm{d}z$
(e) $\int \frac{\sin 2x}{\sin^2 x+3}\,\mathrm{d}x$

Find an antiderivative of function

$$\operatorname{arccotg}\frac{1}{x}$$

Compute

(a) $\int x\ln x\,\mathrm{d}x$
(b) $\int \cos^4 x\,\mathrm{d}x$. Hint: use the formula for $\cos 2x$.