

Homeworks for 13th and 14th week

1. Decide whether the following vectors are linear independent

(a) $\mathbf{u}_1 = (1, -1, 3)$, $\mathbf{u}_2 = (2, 0, 4)$, $\mathbf{u}_3 = (3, 1, 5)$

(b) $\mathbf{u}_1 = (1, 3, 0)$, $\mathbf{u}_2 = (2, 1, 1)$, $\mathbf{u}_3 = (0, 1, -1)$

(c) $\vec{u} = (2, -3, 5)$, $\vec{v} = (1, 0, -2)$, $\vec{w} = (2, -1, 4)$

2. Find the rank of following matrices

(a) $\begin{pmatrix} 0 & 2 & 0 \\ 1 & 2 & -5 \\ -2 & 3 & 6 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & 5 & -3 \\ 3 & 7 & 2 \\ 0 & 1 & -4 \end{pmatrix}$

3. Compute the determinants

(a) $\begin{pmatrix} 5 & 9 \\ 3 & 4 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & -2 & 1 \\ -4 & 5 & -2 \\ 5 & -3 & 1 \end{pmatrix}$

4. Using Frobenius Theorem decide whether the system of linear equation has a solution, find it and describe the set of solutions

(a)

$$x + 2y - z = 1$$

$$2x + 3y + z = 2$$

$$x + 3y - 2z = 1$$

(b)

$$2x + y + 3z = 1$$

$$y + 2z = 0$$

$$x + 2y + z = 4$$

(c)

$$x + 2y + z = -1$$

$$2x - y + 2z = 3$$

$$-x - y - z = 0$$