MAD - worksheet 1

1. Matrices and vectors

Without software compute.

1. Consider matrices $\mathbf{A}$ and $\mathbf{B}$. Compute $\mathbf{A}.\mathbf{B}$, $\mathbf{B}.\mathbf{A}$ and $\mathbf{A}^2 := \mathbf{A}.\mathbf{A}$, if it exists. Compute the determinant of $\mathbf{A}$.

$$\mathbf{A} = \begin{pmatrix} 0 & 2 & 0 \\ 1 & 2 & -5 \\ -2 & 3 & 6 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 4 & -5 \\ -2 & 3 \end{pmatrix}.$$

2. Solve the equation with unknown matrix X

$$\mathbf{X}\,\mathbf{A} - \mathbf{X} = 3\mathbf{B},$$

where

$$\mathbf{A} = \begin{pmatrix} 2 & 2 \\ -1 & 2 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$

3. Consider coefficient matrix $\mathbf{A}$. Verify that there exists an inverse matrix $\mathbf{A}^{-1}$ and using A^{-1} solve the system of linear algebraic equations

$$(\mathbf{A} \mid \vec{b}) = \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & 3 & 1 & 2 \\ 1 & 3 & -2 & 1 \end{array}\right).$$

4. For

   (a)

$$\mathbf{A} = \begin{pmatrix} 2 & 2 & 0 \\ -3 & -3 & 5 \\ 0 & -\frac{1}{4} & 2 \end{pmatrix},$$

   (b)

$$\mathbf{B} = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{pmatrix}$$

   find all eigenvalues.

5. Consider matrix
$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

   (a) Find all eigenvalues of $\mathbf{A}$. Is $\mathbf{A}$ positively semidefinite? For the smallest eigenvalue find the eigenvector.
   (b) Consider linear mapping L which has matrix representation $\mathbf{A}$. Find the kernel of the mapping, determine its dimension and the image of the vector $(1, 0, 2)$. Is L surjective?

6. Consider that matrix $\mathbf{A}$ is 3×3 and its egenvalues are $\lambda_1 = 0$, $\lambda_2 = 3$ and $\lambda_3 = 1$. Compute $\det A$ and trace of $\mathbf{A}$. Is the matrix $\mathbf{A}$ pozitively definite?