

Úlohy k procvičení 1

1. Compute the limits

(a)

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos 2x}$$

(b)

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{\sqrt{x+2} - \sqrt{2}}$$

(c)

$$\lim_{x \rightarrow 0} \frac{x - \sin x}{\tan 2x}$$

2. Decide whether the function f is continuous on $\mathcal{D}(f)$, if there exists the inverse function (sketch the graph), compute the derivative at $x_0 = 0$ (if it exists) and find the equation of the tangent line at point $(1, ?)$.

$$f(x) = \begin{cases} -\sqrt[3]{x} + 1, & x \leq 0, \\ -\frac{1}{(x+1)^3}, & x > 0. \end{cases}$$

3. Find the first derivative

$$f(x) = \ln(-x) + e^{2x} \cos\left(\frac{\pi}{4} - x\right).$$

4. Investigate behavior of the function

$$f(x) = \frac{e^x}{x^2}.$$