

(I) Najděte rovnice tečny ke grafu funkce procházejícího daným bodem A :

$$f(x) = \ln x, \quad A = (1, f(1)),$$

$$g(x) = \frac{1+x}{x-1}, \quad A = (2, g(2)),$$

$$h(x) = (\ln x)^{\ln x}, \quad A = (e, h(e)),$$

$$i(x) = \sin x - x, \quad A = (\pi, i(\pi)),$$

$$j(x) = \frac{2x}{1+\sqrt{x}}, \quad A = (0, j(0)),$$

$$k(x) = \frac{\ln(\cos x)}{\operatorname{tg} x}, \quad A = \left(\frac{\pi}{4}, k\left(\frac{\pi}{4}\right)\right).$$

Výsledky: $t_f : y = x - 1$, $t_g : y = 7 - 2x$, $t_h : y = \frac{1}{e}x$, $t_i : y = \pi - 2x$, $t_j : y = 2x$,
 $t_k : y = -\frac{1}{2} \ln 2 + (-1 + \ln 2) \left(x - \frac{\pi}{4}\right)$

(II) Spočtěte limity následujících funkcí:

$$(i) \lim_{x \rightarrow 1} \frac{\ln^2 x}{x-1}$$

$$(ii) \lim_{x \rightarrow 6} \frac{\sqrt{x-2} - 2}{x-6}$$

$$(iii) \lim_{x \rightarrow +\infty} \frac{\ln(x^2 - 8)}{x^2 - 3x}$$

$$(iv) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

$$(v) \lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\operatorname{tg} x}$$

$$(vi) \lim_{x \rightarrow +\infty} x(\pi - 2 \arctg x)$$

$$(vii) \lim_{x \rightarrow +\infty} \frac{e^x}{e^{2x} + 1}$$

$$(viii) \lim_{x \rightarrow +\infty} \left(\cos \frac{1}{x}\right)^{x^2}$$

$$(ix) \lim_{x \rightarrow +\infty} (\ln x - 2^x)$$

$$(x) \lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 + 1}}{x}$$

$$(xi) \lim_{x \rightarrow 0} (1 - \cos x) \cotg x$$

$$(xii) \lim_{x \rightarrow 0-} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$$

$$(xiii) \lim_{x \rightarrow +\infty} \frac{\ln x}{\sqrt{x}}$$

$$(xiv) \lim_{x \rightarrow +\infty} x^3 2^{-x}$$

$$(xv) \lim_{x \rightarrow 1} (1-x) \operatorname{tg} \frac{\pi x}{2}$$

$$(xvi) \lim_{x \rightarrow 0+} \arcsin x \cotg x$$

$$(xvii) \lim_{x \rightarrow +\infty} \left(\frac{x^2}{x+5} - \frac{x^2}{x-4}\right)$$

$$(xviii) \lim_{x \rightarrow 0} \frac{2^{-x} - 1}{x}$$

$$(xix) \lim_{x \rightarrow 8} \frac{\sqrt[3]{x} - 2}{\sqrt{x} - 2\sqrt{2}}$$

$$(xx) \lim_{x \rightarrow +\infty} \frac{x+1}{\sqrt{x-1}+2}$$

Výsledky

$$(i) 0, (ii) \frac{1}{4}, (iii) 0, (iv) \frac{1}{2}, (v) 1, (vi) 2, (vii) 0, (viii) \frac{1}{\sqrt{e}}, (ix) -\infty, (x) -2, (xi) 0,$$

$$(xii) +\infty, (xiii) 0, (xiv) 0, (xv) \frac{2}{\pi}, (xvi) 1, (xvii) -9, (xviii) -\ln 2, (xix) \frac{\sqrt{2}}{3},$$

$$(xx) +\infty$$