

Integrace racionálních funkcí

$$\begin{array}{ll}
 \text{(i)} \quad \int \frac{5x+2}{x^2+6x+9} dx & \text{(vii)} \quad \int \frac{3x-2}{x^4-x^3} dx \\
 \text{(ii)} \quad \int \frac{x^2-2}{x^2+x-2} dx & \text{(viii)} \quad \int_1^e \frac{2 \ln x}{x(\ln^2 x - 4)} dx \\
 \text{(iii)} \quad \int \frac{x+1}{x^2+2x+3} dx & \text{(ix)} \quad \int_0^1 \frac{3x}{2+x-x^2} dx \\
 \text{(iv)} \quad \int \frac{3x}{2x^2+x-1} dx & \text{(x)} \quad \int \left(\frac{x+1}{x-1} \right)^2 \frac{1}{x} dx \\
 \text{(v)} \quad \int \frac{3e^{2x}}{e^{4x}+e^{2x}-2} dx & \text{(xi)} \quad \int \frac{3x^2}{1-x^3} dx \\
 \text{(vi)} \quad \int_0^1 \frac{1}{2x^2-2x+1} dx & \text{(xii)} \quad \int_{-1}^0 \frac{2x}{(1-x)^3} dx
 \end{array}$$

Výsledky: ($C \in \mathbb{R}$ níže je integrační konstanta)

$$\begin{array}{l}
 \text{(i)} \quad 5 \ln |x+3| + \frac{13}{x+3} + C, \quad x \in (-\infty, 3), (3, +\infty); \quad \text{(ii)} \quad x - \frac{2}{3} \ln |x+2| - \frac{1}{3} \ln |x-1| + C, \\
 x \in (-\infty, -2), (-2, 1), (1, +\infty); \quad \text{(iii)} \quad \frac{1}{2} \ln(x^2+2x+3) + C, \quad x \in \mathbb{R}; \\
 \text{(iv)} \quad \frac{1}{2} \ln |2x-1| + \ln |x+1| + C, \quad x \in (-\infty, -1), (-1, 1/2), (1/2, +\infty); \quad \text{(v)} \quad \frac{1}{2 \ln \frac{|e^{2x}-1|}{e^{2x}+2}} + C, \\
 x \in (-\infty, 0), (0, +\infty); \quad \text{(vi)} \quad \frac{\pi}{2}; \quad \text{(vii)} \quad -\frac{1}{x^2} + \frac{1}{x} + \ln \left| \frac{x-1}{x} \right| + C, \quad x \in (-\infty, 0), (0, 1), (1, +\infty); \\
 \text{(viii)} \quad \ln \frac{3}{4}; \quad \text{(ix)} \quad \ln 2; \quad \text{(x)} \quad \frac{4}{1-x} + \ln |x| + C, \quad x \in (-\infty, 0), (0, 1), (1, +\infty); \\
 \text{(xi)} \quad -\ln |1-x^3| + C, \quad x \in (-\infty, 1), (1, +\infty); \quad \text{(xii)} \quad -\frac{1}{4}
 \end{array}$$