Topics for oral exam – Mathematics B

1. Vectors and matrices, matrix arithmetic. Linear independence of vectors and rank of a matrix.

2. Systems of linear algebraic equations. Consistent and inconsistent systems. Rouché–Capelli theorem.

3. Determinant of a matrix and its properties. Relation between determinant and rank of matrix.

4. Inverse matrices, methods of matrix inversion. Simple matrix equations.

5. Geometry in R^n, especially R^3. Dot product, cross product. Parametric equations of line and plane in R^3. Equation of a plane in R^3.

6. Euclidean space, metric, norm, neighbourhood, open and closed sets, boundary, domain, convex sets, path-connected sets.

7. Functions of several variables. Domain of definition, graph. Contour lines for function of two variables. Continuity and limits for functions of several variables.

8. Partial derivatives, gradient, directional derivative. Total differential, tangent plane and Taylor polynomial of functions of two variables.

9. Newton method for system of two equations.

10. Derivative of mapping, Jacobian matrix, derivatives of composite functions, chain rule.

11. Local extrema of functions of two variables. Stationary points. Hessian matrix. Saddle points.

12. Least square method and its derivation.

13. Implicit functions of a single and several variables, derivatives of implicit functions.

14. Parametric curves in plane and space. Tangent vector to a curve, smooth curve, orientation and a sum of curves.

15. Vector field in the plane and space. Curve integral of a vector field its basic properties and physical interpretation. Work done by force.

16. Path independence of the curve integral of a vector field. Scalar potential of conserative vector field. Differential forms and their integrals.

17. Double integral and its geometrical meaning. Fubini theorem.

18. Substitution for double integral. Polar coordinates. Improper double integral. Evaluation of Gaussian integral.

19. Systems of two first order differential equations. Autonomous and non-autonomous systems, stationary solutions. Predator-prey model. Euler method for initial value problem with system of two first order differential equations.

20. Solving autonomous systems of differential equations with constant coefficients. Eigenvalues and eigenvectors of matrix.