$\boxed{7/1}$

a) $f(x,y) = 3x^2y^2 - 3y + y^3$ $\qquad D(f) = \mathbb{R}^2$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= 6xy^2 = 0 \\ \frac{\partial f}{\partial y} &= 6x^2y - 3 + 3y^2 = 0 \end{aligned}\right\} \longrightarrow 6xy^2 = 0$$

$$\Downarrow$$

$$x = 0 \lor y = 0$$

$$\text{I} \qquad \text{II}$$

I $x = 0$, $\frac{\partial f}{\partial y} = -3 + 3y^2 = 0$

$y^2 = 1$

$y = \pm 1$

$A = [0, 1]$

$B = [0, -1]$

II $y = 0$, $\frac{\partial f}{\partial y} = -3 \neq 0$

DVA STACIONÁRNÍ BODY A, B

$$\frac{\partial^2 f}{\partial x^2} = 6y^2$$

$$\frac{\partial^2 f}{\partial x \partial y} = 12xy$$

$$\frac{\partial^2 f}{\partial y^2} = 6x^2 + 6y$$

$$H_f(x,y) = \det\begin{pmatrix} 6y^2 & 12xy \\ 12xy & 6x^2 + 6y \end{pmatrix} = 36 \cdot \det\begin{pmatrix} y^2 & 2xy \\ 2xy & x^2 + y \end{pmatrix}$$

A: $H_f(0,1) = 36 \det\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 36 > 0 \to$ lok. extrém

$\frac{\partial^2 f}{\partial x^2} > 0 \to$ LOKÁLNÍ MINIMUM V A

$f(0,1) = -3 + 1 = -2$

B: $H_f(0,-1) = 36 \det\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = -36 < 0 \to$ SEDLOVÝ BOD V B

$f(0,-1) = 3 - 1 = 2$

$\boxed{7|1}$

b) $f(x,y) = \frac{y^2 x}{2} + y - \operatorname{arctg} x \qquad D(f) = \mathbb{R}^2$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= \frac{y^2}{2} - \frac{1}{1+x^2} = 0 \\ \frac{\partial f}{\partial y} &= y \cdot x + 1 = 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} \frac{y^2}{2} &= \frac{1}{1+x^2} \\ yx &= -1 \end{aligned}\right\} \rightarrow y = -\frac{1}{x}$$

$$\frac{\left(-\frac{1}{x}\right)^2}{2} = \frac{1}{1+x^2}$$

$$\frac{1}{2x^2} = \frac{1}{1+x^2}$$

$$1+x^2 = 2x^2$$

$$1 = x^2$$

$$x = \pm 1 \quad ; \quad y = -\frac{1}{x} \quad ; \quad A = [1,-1] \quad B = [-1,1]$$

DVA STACIONÁRNÍ BODY A,B

$$\frac{\partial^2 f}{\partial x^2} = \frac{+2x}{(1+x^2)^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = y$$

$$\frac{\partial^2 f}{\partial y^2} = x$$

$$H_f(x,y) = \det\begin{pmatrix} \frac{2x}{(1+x^2)^2} & y \\ y & x \end{pmatrix} = \frac{2x^2}{(1+x^2)^2} - y^2$$

$$H_f(1,-1) = H_f(-1,1) = \frac{2}{4} - 1 = \frac{1}{2} - 1 < 0$$

A,B – SEDLOVÉ BODY

$$f(1,-1) = \frac{1}{2} - 1 - \operatorname{arctg} 1 = -\frac{1}{2} - \frac{\pi}{4} = \frac{-2-\pi}{4}$$

$$f(-1,1) = -\frac{1}{2} + 1 - \operatorname{arctg}(-1) = \frac{1}{2} + \frac{\pi}{4} = \frac{2+\pi}{4}$$

7(1 c)

$$f(x,y) = -x^2 + y - e^{-2x+y}$$

$D(f) = \mathbb{R}^2$

$$\frac{\partial f}{\partial x} = -2x + 2e^{-2x+y} = 0$$

$$\frac{\partial f}{\partial y} = 1 - e^{-2x+y} = 0 \quad \rightarrow \quad 1 = e^{-2x+y}$$

$$0 = -2x + y$$

$$\leftarrow \quad y = 2x$$

$$-2x + 2 \cdot e^{-2x+y} = 0$$

$$-2x + 2 \cdot e^{-2x+2x} = 0$$

$$-2x + 2 = 0$$

$$x = 1 \quad ; \; y = 2x = 2$$

JEDINÝ STACIONÁRNÍ BOD $[1,2]$

$$\frac{\partial^2 f}{\partial x^2}(x,y) = -2 - 4e^{-2x+y} \qquad \frac{\partial^2 f}{\partial x^2}(1,2) = -2-4 = -6$$

$$\frac{\partial^2 f}{\partial x \partial y}(x,y) = 2 \cdot e^{-2x+y} \qquad \frac{\partial^2 f}{\partial x \partial y}(1,2) = 2$$

$$\frac{\partial^2 f}{\partial y^2}(x,y) = -e^{-2x+y} \qquad \frac{\partial^2 f}{\partial y^2}(1,2) = -1$$

$$H_f(1,2) = \det\begin{pmatrix} -6 & 2 \\ 2 & -1 \end{pmatrix} = 6 - 4 > 0 \rightarrow \text{LOK. EXTRÉM}$$

$$\frac{\partial^2 f}{\partial x^2} < 0 \rightarrow \text{LOK. MAXIMUM}$$

$$f(1,2) = -1 + 2 - 1 = 0$$

712

t_i	10	20	30
G_i	28	25,5	-17

TŘI MĚŘENÍ

$m = 3$

$$\sum_{i=1}^{3} t_i = 10 + 20 + 30 = 60$$

$$\sum_{i=1}^{3} (t_i)^2 = 100 + 400 + 900 = 1400$$

$$\sum_{i=1}^{3} t_i \cdot G_i = 280 + 510 - 510 = 280$$

$$\sum_{i=1}^{3} G_i = 28 + 25{,}5 - 17 = 36{,}5$$

$$\begin{pmatrix} 1400 & 60 \\ 60 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 280 \\ 36{,}5 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 1400 & 60 & 280 \\ 60 & 3 & 36{,}5 \end{array}\right) \begin{array}{l} \cdot \frac{1}{20} \\ \cdot 7 \end{array} \sim \left(\begin{array}{cc|c} 70 & 3 & 14 \\ 420 & 21 & 255{,}5 \end{array}\right) -6I \sim \left(\begin{array}{cc|c} 70 & 3 & 14 \\ 0 & 3 & 171{,}5 \end{array}\right)$$

$$70a + 3b = 14$$

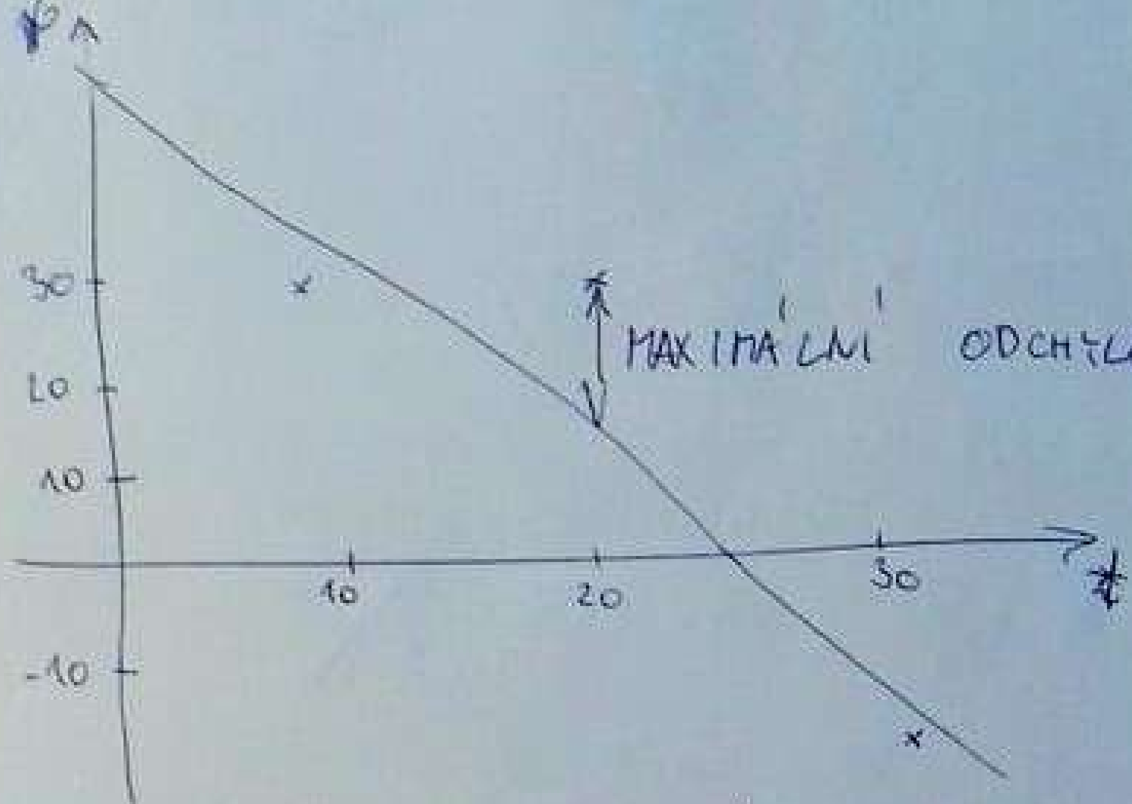
$$70a = 14 - 3b$$

$$a = \frac{14 - 171{,}5}{70} = -2{,}25$$

$$3b = 171{,}5$$

$$b \doteq 57{,}2$$

$$\boxed{G(t) \doteq -2{,}25t + 57{,}2}$$



$$|G_1 - G(t_1)| = |28 - (57{,}2 - 2{,}25 \cdot 10)| \doteq 6{,}7$$

$$|G_2 - G(t_2)| = |25 - (57{,}2 - 2{,}25 \cdot 20)| = \boxed{13{,}3}$$

$$|G_3 - G(t_2)| = |-17 - (57{,}2 - 2{,}25 \cdot 30)| = 6{,}7$$

PRO TROCHU VĚTŠÍ <u>b</u> BY BYLA (PŘI STEJNÉM a) MAXIMÁLNÍ ODCHYLKA MENŠÍ SOUČET KVADRÁTŮ ODCHYLEK BY SE VŠAK ZVĚTŠIL.