2|1

$$\begin{pmatrix} 2 & -1 & -2 & 1 \\ 3 & -2 & 0 & 2 \\ -2 & 2 & -4 & 0 \end{pmatrix} \cdot\tfrac{1}{2} \sim \begin{pmatrix} -1 & 1 & -2 & 0 \\ 3 & -2 & 0 & 2 \\ 2 & -1 & -2 & 1 \end{pmatrix} \begin{matrix} \\ +3\,\mathrm{I} \\ +2\,\mathrm{I} \end{matrix} \sim$$

$$\sim \begin{pmatrix} -1 & 1 & 2 & 0 \\ 0 & 1 & -6 & 2 \\ 0 & 1 & -6 & 1 \end{pmatrix} -\mathrm{II} \sim \begin{pmatrix} -1 & 1 & -2 & 0 \\ 0 & 1 & -6 & 2 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$h(A)=3$, $n=4$, $K=4-3=1$

$$-x_1+x_2-2x_3=0$$
$$x_2-6x_3+2x_4=0$$
$$-x_4=0$$

$x_3=t$, $t\in\mathbb{R}$

$x_2=6t$

$x_1=x_2-2x_3=6t-2t=4t$

$$\vec{x}=t\cdot(4,6,1,0)^T,\ t\in\mathbb{R}$$

2|2

$$\left(\begin{array}{ccc|c} -1 & 1 & -1 & 1 \\ 1 & p & -1 & 1 \\ p & 1 & -1 & 1 \end{array}\right) \begin{matrix} \\ +\mathrm{I} \\ +p\mathrm{I} \end{matrix} \sim \left(\begin{array}{ccc|c} -1 & 1 & -1 & 1 \\ 0 & p+1 & -2 & 2 \\ 0 & p+1 & -1-p & 1+p \end{array}\right) \sim$$

(PRO $p+1\neq 0$, III / $p+1$)

I $p=-1$

$$\left(\begin{array}{ccc|c} -1 & 1 & -1 & 1 \\ 0 & 0 & -2 & 2 \end{array}\right)$$

$h(A)=2$, $n=3$, NEK. MNOHO ŘEŠENÍ

$-x+y-z=1$

$-2z=2 \Rightarrow z=-1$

$y=t$

$x=y-z-1=t$

$\vec{x}=(t,t,-1)^T,\ t\in\mathbb{R}$

$$\sim \left(\begin{array}{ccc|c} -1 & 1 & -1 & 1 \\ 0 & p+1 & -2 & 2 \\ 0 & 1 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 1 & -1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & p+1 & -2 & 2 \end{array}\right) \begin{matrix} \\ -\mathrm{II} \\ -(p+1)\mathrm{II} \end{matrix} \sim \left(\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & p-1 & 1-p \end{array}\right)$$

II $p=1$

$$\left(\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right)$$

$h(A)=2$, $n=3$, NEK. MNOHO ŘEŠENÍ

$-x=0$, $z=t$

$y-z=1$, $y=1+t$

$\vec{x}=(0,1+t,t)^T,\ t\in\mathbb{R}$

III $p\neq 1 \wedge p\neq -1$ $\quad (p\in\mathbb{R}\setminus\{\pm 1\})$

$$\left(\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$h(A)=3$, $n=3$, JEDINÉ ŘEŠENÍ

$-x=0$

$y-z=1$

$z=-1$

$\vec{x}=(0,0,-1)^T$

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$$\alpha(2,-1,3)+\beta(8,-2,-4)+\gamma(4,1,2)=(2,-4,7)$$

$$\left(\begin{array}{ccc|c} 2 & 8 & 4 & 2 \\ -1 & -2 & 1 & -4 \\ 3 & -4 & 2 & 7 \end{array}\right)\cdot\frac{1}{2}\sim\left(\begin{array}{ccc|c} -1 & -2 & 1 & -4 \\ 1 & 4 & 2 & 1 \\ 3 & -4 & 2 & 7 \end{array}\right)+I\sim$$

$$+3I$$

$$\left(\begin{array}{ccc|c} -1 & -2 & 1 & -4 \\ 0 & 2 & 3 & -3 \\ 0 & -10 & 5 & -5 \end{array}\right)\sim\left(\begin{array}{ccc|c} -1 & -2 & 1 & -4 \\ 0 & 2 & 3 & -3 \\ 0 & -2 & 1 & -1 \end{array}\right)+II\sim\left(\begin{array}{ccc|c} -1 & -2 & 1 & -4 \\ 0 & 2 & 3 & -3 \\ 0 & 0 & 4 & -4 \end{array}\right)$$

$4\gamma=-4 \quad \gamma=-1$

$2\beta-3=-3;\ \beta=0$

$\alpha=+4+2\beta+\gamma=+4+1=5$

$$+5\cdot(2,-1,3)+0\,(8,-2,-4)+(-1)\cdot(4,1,2)=(2,-4,7)$$

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$A\cdot\vec{x}=\vec{0}$

1) $$A=\left(\begin{array}{cccc} 1 & 3 & -2 & 0 \\ -2 & 0 & 1 & 1 \\ 1 & -1 & 2 & 2 \end{array}\right)+2I\sim -I$$

$$\sim\left(\begin{array}{cccc} 1 & 3 & -2 & 0 \\ 0 & 6 & -3 & 1 \\ 0 & -4 & 4 & 2 \end{array}\right)\sim\left(\begin{array}{cccc} 1 & 3 & -2 & 0 \\ 0 & -2 & 2 & 1 \\ 0 & 6 & -3 & 1 \end{array}\right)\cdot 3\sim$$

$$\sim\left(\begin{array}{cccc} 1 & 3 & -2 & 0 \\ 0 & -6 & 6 & 3 \\ 0 & 6 & -3 & 1 \end{array}\right)+II\sim\left(\begin{array}{cccc} 1 & 3 & -2 & 0 \\ 0 & -2 & 2 & 1 \\ 0 & 0 & 3 & 4 \end{array}\right)$$

$x_4=3t$

$x_3=-4t$

$2x_2=2x_3+x_4=-8t+3t=-5t$

$x_2=-\frac{5}{2}t$

$x_1=2x_3-3x_2=-8t+\frac{15}{2}t=-\frac{t}{2}$

$$\vec{x}=t\cdot\left(-\frac{1}{2},-\frac{5}{2},-4,3\right)^T$$

$t\in\mathbb{R}$