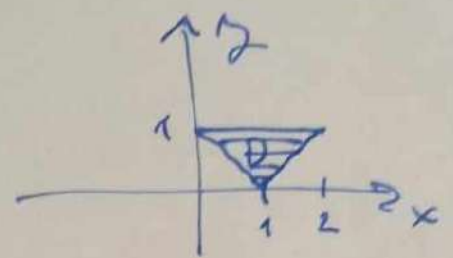


13/1

$$\iint_D \frac{x^2}{1+y^2} dx dy = \int_0^1 x^2 dx \int_0^1 \frac{1}{1+y^2} dy = \left[\frac{x^3}{3} \right]_0^1 \left[\arctan y \right]_0^1 = \frac{\pi}{12}$$

13/2

$$\iint_D e^{x-y} dx dy =$$



$$= \int_0^1 \left(\int_{1-y}^{1+y} e^{x-y} dx \right) dy =$$

$x > 1-y$ $y > 1-x$
 $y+1 > x$ $x > 1-y$

$$= \int_0^1 \left[e^{x-y} \right]_{x=1-y}^{x=1+y} dy = \int_0^1 e - e^{1-2y} dy =$$

$$= \left[e \cdot y + \frac{1}{2} e^{1-2y} \right]_0^1 = e + \frac{1}{2} e^{-1} - \frac{1}{2} e = \frac{1}{2} (e + e^{-1})$$

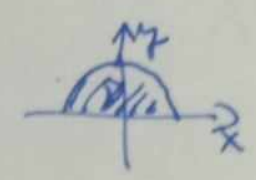
(2. PŮŘADÍ)

$$\iint_D e^{x-y} dx dy = \int_0^1 \left(\int_{x-1}^1 e^{x-y} dy \right) dx + \int_1^2 \left(\int_{1-x}^1 e^{x-y} dy \right) dx$$

13/3

$$D = \{(x,y) \in \mathbb{R}^2 \mid y \geq 0 \wedge x^2 + y^2 \leq 4\}$$

$$V = \iint_D (y+x^2) dx dy =$$



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$$= \int_0^{\pi/2} \int_0^2 (r \cdot \sin \varphi + r^2 \cos^2 \varphi) \cdot r dr d\varphi =$$

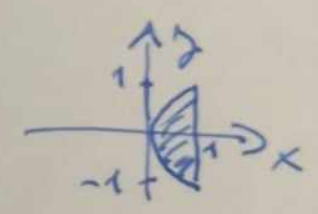
$$= \int_0^{\pi/2} \left[\frac{r^3}{3} \sin \varphi + \frac{r^4}{4} \cos^2 \varphi \right]_{r=0}^2 d\varphi = \int_0^{\pi/2} \left(\frac{8}{3} \sin \varphi + 4 \cos^2 \varphi \right) d\varphi$$

$$= \frac{8}{3} \int_0^{\pi/2} \sin \varphi d\varphi + 2 \int_0^{\pi/2} 1 + \cos(2\varphi) d\varphi = \frac{8}{3} [-\cos \varphi]_0^{\pi/2} + 2 \left[\varphi + \frac{\sin(2\varphi)}{2} \right]_0^{\pi/2} =$$

$$= \frac{8}{3} \cdot (1+1) + 2 \cdot (\pi) = \underline{\underline{\frac{16}{3} + 2\pi}} \quad (j^3)$$

13/5

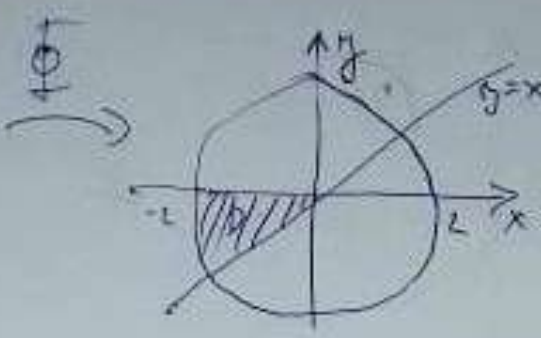
$$\iint_{-1 \leq y \leq 1} \left(\int_0^1 6(\sqrt{x}+y)^5 dx \right) dy =$$



$$= \int_0^1 \left(\int_{-y}^{\sqrt{x}} 6(\sqrt{x}+y)^5 dy \right) dx = \int_0^1 \left[(\sqrt{x}+y)^6 \right]_{y=-\sqrt{x}}^{\sqrt{x}} dx =$$

$$= \int_0^1 (2\sqrt{x})^6 dx = \int_0^1 64 \cdot x^3 dx = \left[\frac{64}{4} x^4 \right]_0^1 = \underline{\underline{16}}$$

$$\boxed{12/1} \iint_D \frac{x^2 y}{x^2 + y^2} dx dy, \quad D = \{(x, y) \in \mathbb{R}^2, x \leq y < 0, x^2 + y^2 \leq 4\}$$

$$H = (0, 2] \times \left(\pi, \frac{5\pi}{4}\right] \xrightarrow{\phi}$$


$$\begin{aligned} \phi \cdot x &= r \cos \varphi \\ y &= r \sin \varphi \\ \sqrt{x^2 + y^2} &= r \end{aligned}$$

$$\iint_D \frac{x^2 y}{x^2 + y^2} dx dy = \iint_H \frac{r^2 \cos^2 \varphi \cdot r \sin \varphi}{r^2 (\cos^2 \varphi + \sin^2 \varphi)} \cdot r dr d\varphi =$$

$$= \int_0^2 \int_{\pi}^{\frac{5\pi}{4}} r^2 \cdot \cos^2 \varphi \cdot \sin \varphi d\varphi dr = \int_0^2 r^2 dr \cdot \int_{\pi}^{\frac{5\pi}{4}} \cos^2 \varphi \sin \varphi d\varphi$$

$$= \left[\frac{r^3}{3} \right]_0^2 \cdot \left[-\frac{\cos^3 \varphi}{3} \right]_{\pi}^{\frac{5\pi}{4}} = \frac{8}{3} \cdot \left(-\frac{1}{3}\right) \cdot \left(\left(-\frac{\sqrt{2}}{2}\right)^3 - (-1)^3 \right)$$

$$= -\frac{8}{9} \cdot \left(1 - \frac{\sqrt{2}}{4}\right) = \frac{8(\sqrt{2}-4)}{9 \cdot 4} = \underline{\underline{\frac{1}{9} \cdot (2\sqrt{2} - 8)}}$$