8|1 $f(x,y) = x^2 \cdot e^{\sqrt{y+1}}$ $\quad D_f = \{(x,y) \in \mathbb{R}^2,\ y \geq -1\}$

$\frac{\partial f}{\partial x}(x,y) = 2x \cdot e^{\sqrt{y+1}}$ $\quad \frac{\partial f}{\partial x}(1,0) = 2 \cdot e$ $\quad f(1,0) = e$

$\frac{\partial f}{\partial y}(x,y) = x^2 \cdot e^{\sqrt{y+1}} \cdot \frac{1}{2\sqrt{y+1}}$ $\quad \frac{\partial f}{\partial y}(1,0) = \frac{1}{2} e$

a) $df(A, dx, dy) = 2e\,dx + \frac{1}{2} e\,dy$

b) $T(x,y) = e + 2e(x-1) + \frac{1}{2} e y$

c) $z - e = 2e(x-1) + \frac{1}{2} e y$

$\boxed{8|2}$ $f(x,y) = \frac{\operatorname{arctg} x}{y}$ $\quad \frac{\operatorname{arctg}(0{,}01)}{0{,}98} = f(0{,}01\,;\,0{,}98)$

$x_0 = 0,\ y_0 = 1$ $\quad f(0,1) = \frac{0}{1} = 0$

$\frac{\partial f}{\partial x}(x,y) = \frac{1}{1+x^2} \cdot \frac{1}{y}$

$\frac{\partial f}{\partial y}(x,y) = -\frac{\operatorname{arctg} x}{y^2}$ $\quad T_2(x,y) = 0 + 1 \cdot (x-0) + 0 + \frac{1}{2} \cdot \left(0 + 2 \cdot (-1) \cdot (x) \cdot (y-1) + \right.$

$= x - x(y-1) = \cancel{x \cdot (1 - y + 1) = x(2-y)}$

$\frac{\partial^2 f}{\partial x^2}(x,y) = \frac{-2x}{(1+x^2)} \cdot \frac{1}{y}$ $\quad \frac{\operatorname{arctg}(0{,}01)}{0{,}98} \doteq T_2(0{,}01\,;\,0{,}98) =$

$\frac{\partial^2 f}{\partial x \partial y}(x,y) = -\frac{1}{(1+x^2) \cdot y^2}$ $\quad = 0{,}01 - 0{,}01 \cdot (-0{,}02) = 0{,}0102$

$\frac{\partial^2 f}{\partial y^2}(x,y) = \frac{+2 \operatorname{arctg} x}{y^3}$

$\boxed{614}$

$$\left.\begin{aligned} 4y^2 - x^2 &= 1 \\ x - e^{y+1} &= 0 \end{aligned}\right\}$$

$4y^2 - x^2 = 1$

$\dfrac{y^2}{\left(\frac{1}{2}\right)^2} - x^2 = 1$

HYPERBOLA

$a = 1$

$b = \frac{1}{2}$ $S = (0,0)$

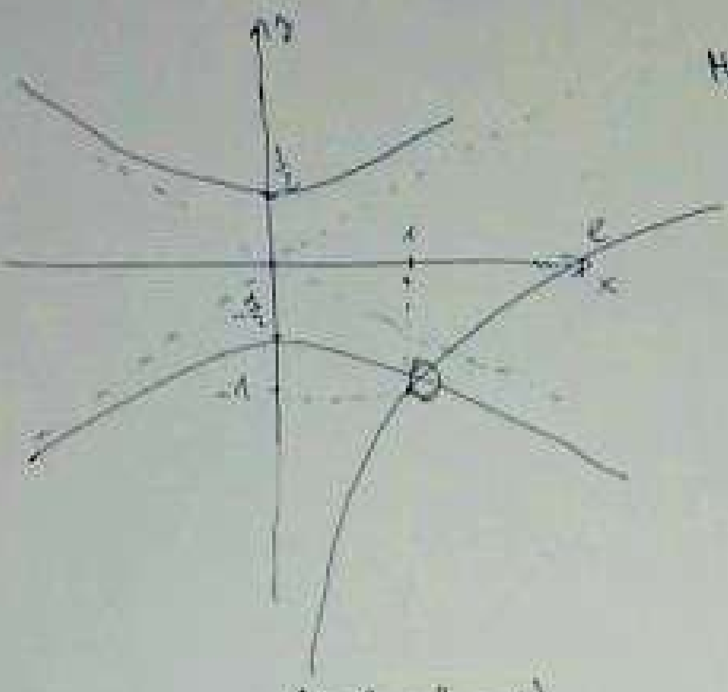
asympt. $y = \pm\frac{1}{2}x$

$x = e^{y+1}$

$\ln x = y + 1$

$y = \ln x - 1$

GRAF FCE



JEDEN PRŮSEČÍK,
SOUSTAVA MÁ PRÁVĚ 1 ŘEŠENÍ

$$F(x,y) = \begin{pmatrix} 4y^2 - x^2 - 1 \\ x - e^{y+1} \end{pmatrix} \qquad F(x_0,y_0) = F(1,-1) = \begin{pmatrix} 4-1-1 \\ 1-1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$J_F(x,y) = \begin{pmatrix} -2x & 8y \\ 1 & -e^{y+1} \end{pmatrix}; \quad J_F(1,-1) = \begin{pmatrix} -2 & -8 \\ 1 & -1 \end{pmatrix}$$

I. ZPŮSOB

$\det J_F = 2 + 8 = 10$

$$J_F^{-1} = \frac{1}{10}\begin{pmatrix} -1 & +8 \\ -1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} - J_F^{-1}(x_0,y_0)\, F(x_0,y_0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \frac{1}{10}\begin{pmatrix} -1 & 8 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \frac{1}{10}\begin{pmatrix} -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 1{,}2 \\ -0{,}8 \end{pmatrix} \qquad \begin{aligned} x_1 &= 1{,}2 \\ y_1 &= -0{,}8 \end{aligned}$$

$\boxed{6/4}$ POKRAČOVÁNÍ

II. ZPŮSOB

$$\begin{pmatrix} J_F \end{pmatrix}\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = -F(x_0, y_0)$$

$$\begin{pmatrix} -2 & -8 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = -\begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$-2\Delta x - 8\Delta y = -2 \qquad 10\Delta y = 2$$

$$\Delta x - \Delta y = 0 \longrightarrow \Delta x = \Delta y \qquad \Delta y = 0{,}2$$

$$x_1 = x_0 + \Delta x = 1 + 0{,}2 = \underline{\underline{1{,}2}}$$

$$y_1 = y_0 + \Delta y = -1 + 0{,}2 = \underline{\underline{-0{,}8}}$$

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c) ANO, NAPŘ.

$$\underline{4(2x-1)^2 - x^2 = 1}, \qquad (y = 2x - 1)$$

NEBO

$$\sqrt{4y^2 - 1} - e^{y+1} = 0 \qquad \left(x = \sqrt{4y^2 - 1}\right)$$

APOD...

7|1

b) $f(x,y) = \frac{y^2 x}{2} + y - \operatorname{arctg} x \qquad D(f) = \mathbb{R}^2$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= \frac{y^2}{2} - \frac{1}{1+x^2} = 0 \\ \frac{\partial f}{\partial y} &= y \cdot x + 1 = 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} \frac{y^2}{2} &= \frac{1}{1+x^2} \\ yx &= -1 \end{aligned}\right\} \to y = -\frac{1}{x}$$

$$\frac{\left(-\frac{1}{x}\right)^2}{2} = \frac{1}{1+x^2}$$

$$\frac{1}{2x^2} = \frac{1}{1+x^2}$$

$$1+x^2 = 2x^2$$

$$1 = x^2$$

$$x = \pm 1 \ ; \ y = -\frac{1}{x} \ ; \quad A = [1,-1] \quad B = [-1,1]$$

DVA STACIONÁRNÍ BODY A,B

$$\frac{\partial^2 f}{\partial x^2} = \frac{+2x}{(1+x^2)^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = y$$

$$\frac{\partial^2 f}{\partial y^2} = x$$

$$H_f(x,y) = \det\begin{pmatrix} \frac{2x}{(1+x^2)^2} & y \\ y & x \end{pmatrix} = \frac{2x^2}{(1+x^2)^2} - y^2$$

$$H_f(1,-1) = H_f(-1,1) = \frac{2}{4} - 1 = \frac{1}{2} - 1 < 0$$

A,B – SEDLOVÉ BODY

$$f(1,-1) = \frac{1}{2} - 1 - \operatorname{arctg} 1 = -\frac{1}{2} - \frac{\pi}{4} = \frac{-2-\pi}{4}$$

$$f(-1,1) = -\frac{1}{2} + 1 - \operatorname{arctg}(-1) = \frac{1}{2} + \frac{\pi}{4} = \frac{2+\pi}{4}$$