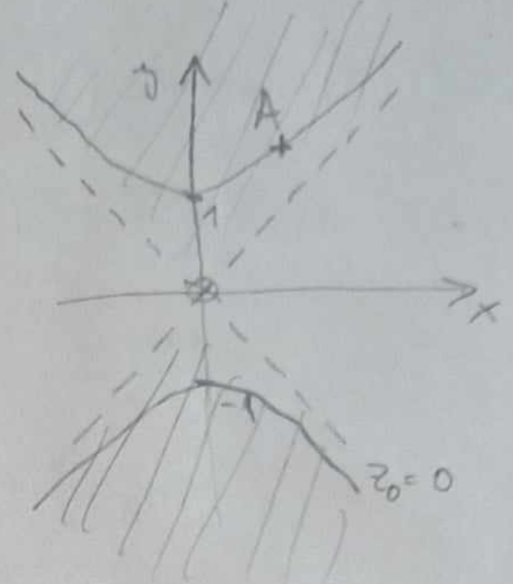
6/1

a) $D_f = \{(x,y)\in\mathbb{R}^2 \mid y^2 > x^2\} = \{(x,y)\in\mathbb{R}^2 \mid |y|>|x|\}$

$z_0 = f(1,\sqrt{2}) = \ln(2-1) = \ln 1 = 0$

$\ln(y^2-x^2) = 0$

$\underline{y^2-x^2=1}$



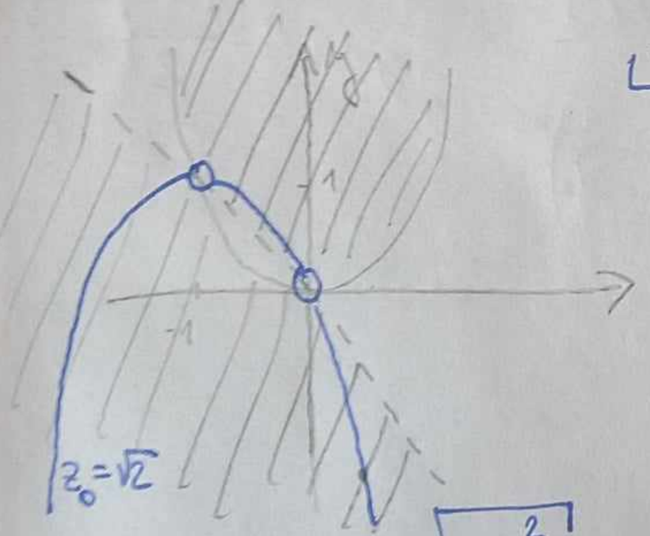
b) $D_f = \left\{(x,y)\in\mathbb{R}^2 \mid x+y \neq 0,\ \frac{y-x^2}{x+y} \geq 0\right\}$

$z_0 = f(1,-3) = \sqrt{\frac{-3-1}{-2}} = \sqrt{2}$

$\sqrt{\frac{y-x^2}{x+y}} = \sqrt{2}$

$y - x^2 = 2x + 2y$

$\underline{-x^2-2x = y \wedge (x,y)\in D_f}$



6/2

a) $\lim\limits_{(x,y)\to(1,1)} \sqrt{\frac{y-x^2}{x+y}} = 0$

f je SPOJITÁ na Df

b) $\lim\limits_{(x,y)\to(1,-3)} f(x,y) = \sqrt{2}$

c) $\lim\limits_{(x,y)\to(0,0)} \sqrt{\frac{y-x^2}{x+y}}$ NEEXISTUJE „K BODU SE BLÍŽÍ" VRSTEVNICE $z_0=\sqrt{2}$ i $z_0=0$

$(y=x^2, (x,y)\in D(f))$

6(3) a) $D_f = \{(x,y) \in \mathbb{R}^2 \mid y \neq 0\}$

b) $z_0 = f(1,1) = \operatorname{arctg}\left(\frac{1+1-1}{1}\right) = \frac{\pi}{4}$

$$\operatorname{arctg}\left(\frac{x+y-y^3}{y}\right) = \frac{\pi}{4}$$

$$\frac{x+y-y^3}{y} = 1$$

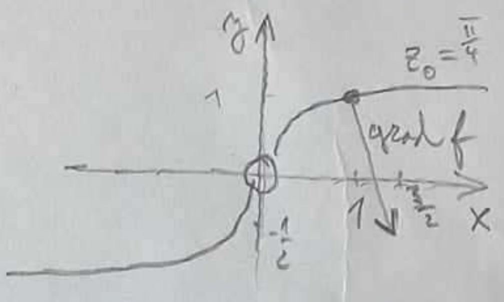
$$x+y-y^3 = y$$

$$\underline{x = y^3 \ \wedge \ y \neq 0}$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{1}{1+\left(\frac{x+y-y^3}{y}\right)^2} \cdot \frac{1}{y} \Rightarrow \frac{\partial f}{\partial x}(1,1) = \frac{1}{1+1} = \frac{1}{2}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{1}{1+\left(\frac{x+y-y^3}{y}\right)^2} \cdot \frac{(1-3y^2)\cdot y - (x+y-y^3)\cdot 1}{y^2} \Rightarrow \frac{\partial f}{\partial y}(1,1) = \frac{1-3-1}{2} = -\frac{3}{2}$$

$$\operatorname{grad} f(A) = \left(\frac{1}{2}, -\frac{3}{2}\right)$$



$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(\frac{y}{y^2+(x+y-y^3)^2}\right) = \frac{y^2+(x+y-y^3)^2 - y\cdot\left(2y+2(x+y-y^3)\cdot(1-3y^2)\right)}{\left(y^2+(x+y-y^3)^2\right)^2}$$

$$\frac{\partial^2 f}{\partial x \partial y}(A) = \frac{1+1-(2+2\cdot(-2))}{4} = \underline{\underline{1}}$$