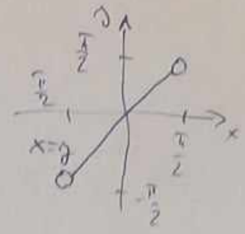
11/1

NAPŘÍKLAD: $x = \operatorname{arctg} t$, $y = \operatorname{arctg} t$, $t \in \mathbb{R}$



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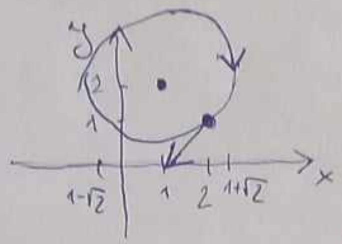
$x = 1 + \sqrt{2}\sin t$
$y = 2 + \sqrt{2}\cos t$, $t \in \langle 0, 2\pi \rangle$

KRUŽNICE $S = [1, 2]$
$r = \sqrt{2}$



$T = (2, 1)$

$1 + \sqrt{2}\sin t = 2$
$2 + \sqrt{2}\cos t = 1$

$\underbrace{\sin t = \frac{\sqrt{2}}{2} \qquad \cos t = -\frac{\sqrt{2}}{2}}$

$\Rightarrow t_0 = \frac{3\pi}{4}$

$\vec{r}'(t) = (\sqrt{2}\cos t, -\sqrt{2}\sin t)$

$\vec{v}_0 = \vec{r}'(t_0) = \left(\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right), -\sqrt{2}\cdot\frac{\sqrt{2}}{2}\right) = (-1, 1)$

$$\int_C -y\,dx + 2x\,dy = \int_0^{2\pi} -(2+\sqrt{2}\cos t)\cdot\sqrt{2}\cos t + 2(1+\sqrt{2}\sin t)(-\sqrt{2}\sin t)\,dt =$$

$$= \sqrt{2}\int_0^{2\pi} -2\cos t - \sqrt{2}\cos^2 t - 2\sin t - 2\sqrt{2}\sin^2 t\,dt =$$

$$= \sqrt{2}\int_0^{2\pi} -2\cos t - 2\sin t - \sqrt{2}\cos^2 t - \sqrt{2}\sin^2 t - \sqrt{2}\sin^2 t\,dt =$$

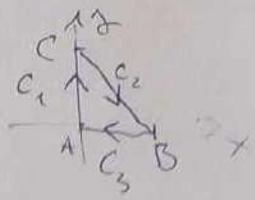
$$= \sqrt{2}\int_0^{2\pi} -2\cos t - 2\sin t - \sqrt{2}\cdot 1 - \sqrt{2}\,\frac{1-\cos(2t)}{2}\,dt =$$

$$= \sqrt{2}\left[-2\sin t + 2\cos t - \sqrt{2}\,t - \frac{\sqrt{2}}{2}t + \sqrt{2}\,\frac{\sin(2t)}{4}\right]_0^{2\pi} =$$

$$= \underline{-6\pi}$$

$\boxed{11/3}$

$C = C_1 + C_2 + C_3$



$C_1: x = 0$ $\qquad dx = 0$
$\quad y = t \quad , t \in \langle 0, 2 \rangle \qquad dy = dt$

$C_2: x = t$ $\qquad dx = dt$
$\quad y = 2 - 2t \quad , t \in \langle 0, 1 \rangle \qquad dy = -2dt$

$C_3: x = 1 - t \quad , t \in \langle 0, 1 \rangle \qquad dx = -dt$
$\quad y = 0 \qquad dy = 0$

$$\int_{C_1} (e^x - y)dx + x^2 y\, dy = \int_0^2 0 + 0\, dt = 0$$

$$\int_{C_2} (e^x - y)dx + x^2 y\, dy = \int_0^1 e^t - (2 - 2t) + t^2 \cdot (2 - 2t) \cdot (-2) dt =$$

$$= \int_0^1 e^t - 2 + 2t - 4t^2 + 4t^3 dt = \left[e^t - 2t + t^2 - \frac{4}{3} t^3 + t^4 \right]_0^1 =$$

$$= e - 2 + 1 - \frac{4}{3} + 1 - 1 =$$

$$= e - \frac{7}{3}$$

$$\int_{C_3} (e^x - y)dx + x^2 y\, dy = \int_0^1 (e^{1-t} - 0)(-1)dt = \left[e^{1-t} \right]_0^1 = 1 - e$$

$$\int_C \vec{F} \cdot \vec{dr} = e - \frac{7}{3} + 1 - e = \underline{\underline{-\frac{4}{3}}}$$